Hypothesis Testing for Population Proportion

BMR 617

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General Steps in a Hypothesis Test

- Check assumptions and write the null and alternative hypotheses (they should be mutually exclusive and exhaustive)
- Calculate a test statistic
- Determine a p-value associated with the test statistic
  - recall what was mentioned in previous lecture about p-values
  - the smaller the p-value, the more that is in favor of the alternative hypothesis
- Choose between the null and alternative hypotheses
- Make your conclusion

Proportion

- Categorical variables
  - Classic example: A balanced coin is flipped 100 times and percentage of heads is 48%.
  - sample proportion ($\hat{p} = \frac{48}{100} = 0.48$)
  - sample proportion → draw conclusion about population proportion
  - How can $\hat{p}$ be an accurate measure of *p* if another sample (100 flips) would give 52 heads?
  - Take many samples of 100 flips → histogram would look normal with mean=50%

Notation

- H_0: null hypothesis
- H_a: alternative hypothesis
- p_0: null hypothesized value (p-not or p-zero)

- H_0: $\hat{p} = p_0$
- H_a: $\hat{p} \neq p_0$ or H_a: $\hat{p} > p_0$ or Ha: $\hat{p} < p_0$ (select only one H_a)

Test statistic

summary of a particular sample that is somehow sensitive to differences between the null and alternative hypotheses

Example Test of a Proportion

- A Marshall University (MU) study finds that 30% of 12th grade females think they are overweight. Is this percent lower for college age females? Let p = proportion of college age females who think they are overweight.
- H_0: $\hat{p}$ = 0.30 (or greater) i.e., no difference from MU study finding
- H_a: $\hat{p} < 0.30$ (proportion thinking they are overweight is less for college age females)

One Proportion Z Test

- Used to compare observed proportion to an expected one (only 2 categories)
- Test statistic is a z-score (z)
- $z = \frac{\hat{p} - p_0}{\sqrt{\frac{\hat{p}\hat{q}}{n}}}$

where:

z = test statistic

n = sample size

$\hat{p}$ = observed proportion

$\hat{q} = 1 - \hat{p}$

p_0 = null hypothesized value (expected proportion; also p_e)

One Proportion Z Test

- Population of TH mice have half male and half female ($\hat{p} = 0.5 = 50\%$). Some of these TH mice (n=200) developed diabetes where 120 are male and 80 are female.
- We want to know whether males are at higher risk than females, i.e., diabetes affects more males than females
- Given:
  - number of successes (male TH mice with diabetes) = 120
  - observed proportion of males ($\hat{p}$) is 120/200
  - observed proportion of females ($\hat{q}$) is 1-$\hat{p}$ = 80/200
  - null hypothesized value (expected proportion) of males (p_0) is 0.5 or 50%
  - number of observations (n) is 200
- Is the observed proportion of males ($\hat{p}$) equal to the expected proportion (p_0)?

One Proportion Z Test

- Is the observed proportion of males ($\hat{p}$) equal to the expected proportion (p_0)?
- H_0: $\hat{p}$= p_0 (same)
- H_a: $\hat{p} \neq p_0$ (different)
- $z = \frac{\hat{p}-p_0}{\sqrt{\frac{\hat{p}\hat{q}}{n}}}$
  - if $|z| < 1.96$, then the difference is not significant at 5%
  - if $|z| \geq 1.96$, then the difference is significant at 5% (remember the previous comment on p-values)
  - confidence interval of $\hat{p}$ at 95% is: $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$

Compute One Proportion Z Test

- `binom.test()` or `prop.test()`
- `binom.test()`: computes exact binomial test (for small sample size)
- `prop.test()`: can be used when n>30 since it uses normal approximation to binomial distribution
- Syntax
  - `binom.test(x, n, p = 0.5, alternative = "two.sided")`
  - `prop.test(x, n, p = NULL, alternative = "two.sided", correct = TRUE)`
  - where
    - x = number of successes = 120
    - n = total number of observations = 200
    - p = null hypothesized value (probability to test against) = 0.5
    - correct: logical containing whether Yates’ continuity correction should be used or not

One Proportion Z Test

- Does diabetes affect more males than females?
- `res <- prop.test(x = 120, n = 200, p = 0.5, correct = FALSE)`
- Print the results:

```
1-sample proportions test without continuity correction

data:  x out of n, null probability p
X-squared = 8, df = 1, p-value = 0.004678
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.5308367 0.6653942
sample estimates:
  p
0.6
```

- If the rate of diabetes were the same in males and females, the probability of observing data at least as extreme as that actually observed is 0.004678